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Hybrid discrete-time modelling and explicit model predictive control for brushed DC motor speed control[1]

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Abstract. The development of an innovative EMPC (Explicit Model Predictive Control) scheme for brushed DC (Direct Current) motor speed control is traced to overcome the control difficulties encountered in practice. Based on multi-parametric linear/quadratic programming, EMPC moves the online iteration algorithm of optimal control problem off-line to reduce the online computation time. By dividing the switching period into subperiods, a hybrid discrete-time model of the brushed DC motor system, which reflects the switching and hybrid nature of the system, is derived for EMPC controller design. The proposed EMPC scheme achieves better performance by coordinate control and the steady state error is eliminated by feedback correction. In addition, EMPC is more suitable for implementation on digital controllers compared with existing continuous controllers. Simulations show the effectiveness of the proposed method compared with conventional ones under unmodelled disturbances.

Key words. Explicit model predictive control, brushed DC motor, speed control.

1. Introduction

With the advantages of torque coefficient, high reliability and excellent overload capacity, brushed DC motors are widely used in industry [1, 2]. The motor control system plays an important role in the smooth and rapid operational performance of the brushed DC motor system. When working at low speed, many problems, such as steady state error, instability, oscillation and so on, can be caused by unmodelled disturbances [3].

In the literature, the speed control of the brushed DC motor has been studied for

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decades, and many methods have been proposed. PID control schemes are widely used in the speed control of the brushed DC motor due to its simplicity and reliability. However in the case of unmodelled disturbances, the performance of the PID is deteriorated [4]. The unmodelled disturbances, such as input source voltage variations, frictions, load torque variations and so on, are major problems remain unsolved in the speed control of the brushed DC motor. Many researchers have proposed new methods for the speed control. The support vector machines are studied in [5] to meet the design requirements. Wiener-type neural network is studied in [6] to improve the speed control performance of the system. An adaptive robust speed control is proposed in [7] based on a disturbance observer. Backstepping-based output feedback controller is studied in [8]. The performance of the above mentioned modern controllers have been verified by simulations and experiments.

Among these modern control strategies, one research direction with significant potential is coordinated control using a MPC (Model Predictive Control) algorithm which have already been proposed in the brushed DC motor control [9]. In the literature, MPC is regarded as an efficient control strategy based on the completely multivariable system framework [10]. There are several aspects, such as the ability to perform optimization and constraint handling, make MPC strategy attractive to both practitioners and academics. Relying on a dynamic model of the process, the traditional MPC schema uses a receding horizon control principle and the optimal control problem is solved by on-line iteration. As a result, the application of MPC strategy needs expensive on-line computation power and MPC is labeled as a technology for slow processes. Recently, EMPC is proposed to handle this problem [11–13]. EMPC moves all the computations necessary for the implementation of MPC off-line using multi-parameter programming, while preserving all its other characteristics. EMPC divide the state space into critical regions off-line and for each critical region the optimal control law is fixed and given. Therefore, EMPC reduces on-line computation time and renders MPC suitable for fast systems such as switched power converters. For power electronics, EMPC has been studied in some electrical drives [14]. As for brushed DC motor speed control, the application of EMPC strategies is still under investigation.

According to the circuit topology and switching philosophy, this paper proposes a hybrid discrete-time modelling method for brushed DC motor speed control which is simple and adequate as a predictive model. Based on the discrete-time model, EMPC strategy is developed to reduce the on-line computation time and regulate the output voltage under a wide range of operating conditions. As a result, the dynamic performance is developed and the complexity of controller is greatly reduced [15].

Besides these benefits, the proposed EMPC respects all the constraints of the brushed DC motor speed control system including the current constraint of the armature, the input constraint, the input rate constraint, which is difficult to handle in the conventional control strategies.

2. Physical structure and mathematical model

2.1. Physical structure

An overview of the brushed DC motor is given in this section to illustrate the background of the control problem. Fig. 1 depicts the circuit topology and physical setup of the brushed DC motor where ω represents the angular speed, u denotes the input voltage, L, i, R, and E are equivalent inductance, current, equivalent resistance, back-EMF of armature respectively. Symbol J denotes the total inertia of the system, f is the friction coefficient, E_s and i_s are the excitation voltage and current, respectively.

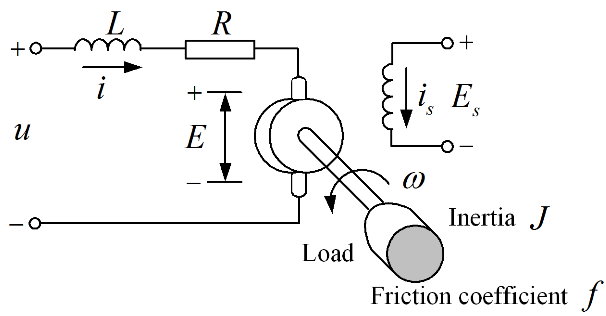

Fig. 1. Physical setup of brushed DC motor

As stated in introduction, angular speed control of the brushed DC motor is an important topic. There are main three kinds of angular speed control method in practice: change of the input voltage u, tuning of the resistance R, and change of the excitation voltage E_s. The first method is the most frequently used for small and medium brushed DC motors. Changing of the input voltage is usually accomplished by the PWM (Pulse Width Modulation) method. A switch is inserted into the power supply circuit, the power supply is turned on and off periodically with fixed frequency. The averaged input voltage U_d is determined by the percent of T_{on} with respect to switching period T (i.e., T_{on}/T), which is called the duty cycle d. As a result, the angular speed of the brushed DC motor can be controlled by the duty cycle d.

2.2. Mathematical model

Mathematical model of the brushed DC motor can be derived by choosing $x(t) \in [i, \omega]^T$ as the state vector. For each switching period T, the system has different dynamics in T_{on} and T_{off}, which are referred to as different modes. By applying Kirchhoff's Voltage and Current Laws, and Newton's Dynamic Law, we have the continuous time state space model for each mode.

2.2.1. Model 1 (T_{on}).

$$\dot{x} = Fx + g = \begin{bmatrix} -\frac{R}{L} & -\frac{C_{\mathrm{e}}}{L} \\ \frac{C_{\mathrm{M}}}{J} & -\frac{f}{J} \end{bmatrix} \cdot x + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \cdot u\,, \tag{1}$$

$$y = [0\ 1]x\,. \tag{2}$$

2.2.2. Model 2 (T_{off}).

$$\dot{x} = Fx = \begin{bmatrix} -\frac{R}{L} & -\frac{C_{\mathrm{e}}}{L} \\ \frac{C_{\mathrm{M}}}{J} & -\frac{f}{J} \end{bmatrix} \cdot x\,, \tag{3}$$

$$y = [0\ 1]x\,. \tag{4}$$

where C_{e} and C_{M} are the voltage coefficient and torque coefficient of the brushed DC motor, respectively.

For the optimal control problem formulation, a discrete-time prediction model is needed. The model should be beneficial to capture the evolution of the states not only at time instant kT but also within the switching period which would be able to impose system constraints on intermediate samples. This could be done by dividing the period length T into v subperiods T_0, which is shown in Fig. 2 (take $v = 3$ for example).

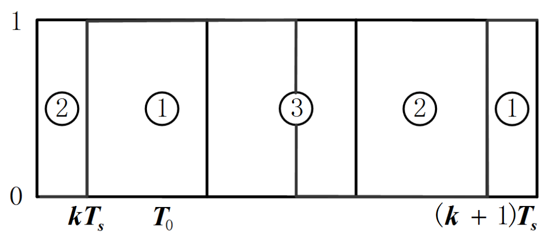

Fig. 2. Position of the switch and active mode in the respective subperiod

According to the switch position and continuous time model, we have different models for the three different modes of subsystems.
Mode 1: $\xi(n+1) = \Phi\xi(n) + \Psi,\quad d(k) \geq (n+1)/v,$
Mode 2: $\xi(n+1) = \Phi\xi(n),\quad d(k) \geq n/v,$
Mode 3: $\xi(n+1) = \Phi\xi(n) + \Psi(vd(k) - n),\quad n/v \leq d(k) \leq (n+1)/v$, where

$$\Phi = \mathrm{e}^{FT_0}\,, \tag{5}$$

$$\Psi = \int_0^{T_0} \mathrm{e}^{F(T_0-t)}\,\mathrm{d}t \cdot g = \int_0^{T_0} \mathrm{e}^{Ft}\,\mathrm{d}t \cdot g\,. \tag{6}$$

The corresponding matrices F and g are given in (1)–(4).

3. Proposed control schema: EMPC

3.1. Hybrid discrete-time modelling

In order to implement the EMPC scheme, an adequate discrete-time model should be derived at first step. The discrete-time model must be accurate enough to guarantee satisfactory performance and sufficiently simple for controller design.

Start from the normalized continuous state space model (1)–(4). Divide the switching period into three equal intervals as shown in Fig. 2. We have the hybrid discrete time state space model of the brushed DC motor in hybrid (piecewise affine) form given as follows

$$x(k+1) = \begin{cases} \Phi^3 x(k) + 3\Phi^2\Psi d(k) & d(k) \in [0, 1/3]. \\ \Phi^3 x(k) + \Phi^2\Psi + 3\Phi\Psi(d(k) - 1/3) & d(k) \in [1/3, 2/3]. \\ \Phi^3 x(k) + \Phi^2\Psi + \Phi\Psi + 3\Psi(d(k) - 2/3) & d(k) \in [2/3, 1]. \end{cases}, \tag{7}$$

where matrices Φ and Ψ are given by (5) and (6).

3.2. EMPC based on multi-parametric programming: a brief review

For the sake of the readers' convenience, a brief review of EMPC based on multi-parametric programming, which has been applied to typical linear quadratic control for constrained systems [15], is given in this section.

Consider a discrete-time MIMO LTI (Linear Time-Invariant) system of the regular form:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \tag{8}$$

subjected to the constraints

$$x_{\min} \leq x(k) \leq x_{\max}, y_{\min} \leq y(k) \leq y_{\max}, u_{\min} \leq u(k) \leq u_{\max} \tag{9}$$

at all time instants $k \geq 0$. In equations (8)-(9), the state vector $x(k) \in \mathrm{R}^n$, output vector $y(k) \in \mathrm{R}^q$, and the input vector $u(k) \in \mathrm{R}^p$, the state space matrices A, B and C are of proper dimension.

In the literature, for system (8), MPC solves the following optimization problem

$$\min_{U \triangleq \{u_k, \cdots, u_{k+N_u-1}\}}$$

$$\left\{ J\left(U, x(k)\right) = x'_{k+N_\mathrm{p}} P x_{k+N_\mathrm{p}} + \sum_{j=0}^{N_\mathrm{p}-1} \left[x'_{k+j} Q x_{k+j} + u'_{k+j} R u_{k+j} \right] \right\} \tag{10}$$

subjected to system dynamics and constraints.

The idea of MPC is the construction of an optimal control input sequence $U^* = \left\{u^*_k,\ u^*_{k+1}, \cdots,\ u^*_{k+N_u-1}\right\}$, which minimizes the cost function J in (10) with respect to the state, output and input constraints (9). And MPC employs the receding horizon control principle, only the first step of the control input U^* (i.e., $u^*(k)$) is taken into the system at the time instant k. As for $k+1$, the whole procedure will be repeated once again, that is, this optimal programming will be taken over and over again along the control time sequence. MPC has been regarded as one of the practical modern control strategies. However, it usually has the drawback of heavy on-line computational burden. As a result, the application of MPC strategy needs expensive on-line computation power and MPC is labeled as a technology for slow processes. In this paper, we adopt a recently proposed EMPC strategy based on multi-parametric programming which is able to move all the computations of MPC off-line. And the resulting controller is an explicit piecewise affine function of the states which is suitable for the discrete-time hybrid model of the brushed DC motor derived in Section 3.1.

By substituting $x_{k+j} = A^j x(k) + \sum_{m=0}^{j-1} A^{j-m-1} B u_{k+m}$, the optimal problem (10) can be rewritten in compact form as

$$V^*(x_k) = \frac{1}{2} x_k^{'} Y x_k + \min_{U} \left\{ \frac{1}{2} U^{'} H U + x_k^{'} F U, \ \text{ s.t. } \ GU \leq W + E x_k \right\}, \tag{11}$$

where $U = \left[u_k^{'}, u_{k+1}^{'}, \cdots, u_{k+N_u-1}^{'} \right]^{'}$ is the optimization vector, and H, F, Y, G, W, E are obtained from Q and R in (10). As proposed in [10], the quadratic problem (11) can be solved by multi-parametric quadratic programming. By setting $z \triangleq U + H^{-1} F^{'} x_k$, (11) can be transformed into the form

$$V_z^*(x_k) = \min_{z} \frac{1}{2} z^{'} H z \ \text{ s.t. } \ Gz \leq W + S x_k \,, \tag{12}$$

where $S \triangleq E + GH^{-1}F^{'}$ and $V_z^*(x_k) = V^*(x_k) - \frac{1}{2} x_k^{'} (Y - FH^{-1}F^{'}) x_k$.

For the multi-parametric quadratic programming of problem (12), we introduce the following result which is the key to construct a piecewise affine state-feedback control law for EMPC.

Lemma 1 [11]: For a quadratic programming problem stated in (12), let $z = z_0^*$ be the optimal solution for a given state x_k^0 and $\left\{ \tilde{G}, \tilde{W}, \tilde{S} \right\}$ is the uniquely determined set of active constraints $\tilde{G} z_0^* = \tilde{W} + \tilde{S} x_k^0$. Assume that the rows of $\tilde{G}$ are linearly independent, and let CR_0 be the set of all vectors x_k for which the combination of constraints $\left\{ \tilde{G}, \tilde{W}, \tilde{S} \right\}$ is active at the optimum (CR_0 being referred to as critical region). Then, the optimal solution z^* of (12) is a uniquely defined affine function of x_k

$$z^* = H^{-1} \tilde{G}^{'} \left(\tilde{G} H^{-1} \tilde{G}^{'} \right)^{-1} \left(\tilde{W} + \tilde{S} x_k \right) \tag{13}$$

over the polyhedral region CR_0 defined by

$$GH^{-1}\tilde{G}'\left(\tilde{G}H^{-1}\tilde{G}'\right)^{-1}\left(\tilde{W}+\tilde{S}x_k\right)\le W+Sx_k^0 \tag{14}$$

and

$$\left(\tilde{G}H^{-1}\tilde{G}'\right)^{-1}\left(\tilde{W}+\tilde{S}x_k\right)\le 0\,. \tag{15}$$

To summarize, multi-parametric quadratic programming systematically subdivides the space X of parameters x_k into critical regions (CRs). For every CR, the optimal solution z^* is an affine function of x_k. Once the critical region CR_0 has been defined, the rest of the space $CR^{\text{rest}} \triangleq X\backslash CR_0$ can be explored and new critical regions will be generated by an iterative algorithm which partition CR^{rest} recursively. As for the iterative algorithm, interested readers may refer to related articles of EMPC [11].

As a result, the state space X is divided into critical regions, and in each region, the optimal solution $z^*(x_k)$ is an affine function of x_k (i.e., $z^*(x_k)$ is piecewise affine) which can be calculated off-line.

3.3. EMPC controller design

A simple brushed DC motor is taken as an example to the implementation of EMPC strategies. Its parameters are given as follows: $u = 12\,\text{V}$, $L = 0.329\,\text{mH}$, $R = 1.4\,\Omega$, $C_e = 0.06\,\text{V·s/rad}$, $C_M = 0.08\,\text{N·m/A}$, $J = 0.0137\,\text{g·m}^2$, and $f = 0.008\,\text{N·m·s/rad}$. The hybrid discrete-time model can easily be derived from these parameters.

The control objectives are to regulate the output voltage to its reference, or in other words, to minimize the output voltage error $\omega_{\text{o,err}} = \omega_{\text{o}} - \omega_{\text{o,ref}}$ with respect to the constraints on the armature current, input duty cycle and input rate. Additionally, we introduce the difference of two consecutive duty cycles

$$\Delta d(k) = d(k) - d(k-1)\,.$$

Define now the penalty matrix $Q = \text{diag}(q_1, q_2)$ and vector $\varepsilon(k) = [\omega_{\text{o,err}}(k), \Delta d(k)]$. The performance index function is given as

$$J = \sum_{k=0}^{N-1} \varepsilon^{\text{T}}(k) Q \varepsilon(k)\,. \tag{16}$$

Based on the piecewise affine discrete-time linear model (7) and performance index function (16), piecewise affine EMPC controllers can be designed according to the standard procedure discussed in Section 3.2.

4. Simulation and experiment results

Dynamic simulations using Matlab were carried out to evaluate the performance of the proposed EMPC strategy. The simulations were performed on a mathematical model which is tuned in agreement with the real plant and adequately grasps the dynamic behavior of the brushed DC motor. Therefore, they are very useful for tasks such as high performance controller design and evaluation.

For the sake of comparison, we also report simulation results of the conventional PI algorithm which adjusts the control input of the PWM according to the difference between the output angular speed and the reference speed. The PI regulators are of the form

$$d(k) = 0.02\left(\omega_{\mathrm{o,ref}} - \omega_{\mathrm{o}}\right) + 0.004 \int \left(\omega_{\mathrm{o,ref}} - \omega_{\mathrm{o}}\right) \mathrm{d}t\,,$$

which were developed by on-line tuning.

As far as the characteristics of the brushed DC motor system are concerned, we choose prediction horizon $N = 6$ for EMPC. The weight matrices are selected as $Q = \mathrm{diag}(0.01, 5)$. The constraints are taken from the parameters $0 \le d \le 1$, $|\Delta d| \le 0.2$. As for system model mismatches and sustaining disturbances, the predictive model will be inadequate, which will cause steady state error. A feedback correction algorithm is proposed to eliminate the steady state error, that is

$$y_p\left(k+i|\,k\right) = y_M\left(k+i|\,k\right) + h_i e(k), \quad i = 1, \cdots, N\,, \tag{17}$$

$$e(k) = y(k) - y_M(k)\,, \tag{18}$$

where $e(k)$ is the prediction error at time instant k, $y_M\left(k+i|\,k\right)$ is the predicted output based on predictive model, h_i is the feedback correction coefficient which can be determined by trade-off or online identification, $y_P\left(k+i|\,k\right)$ is the corrected predictive output, which is finally taken into account in the EMPC strategy.

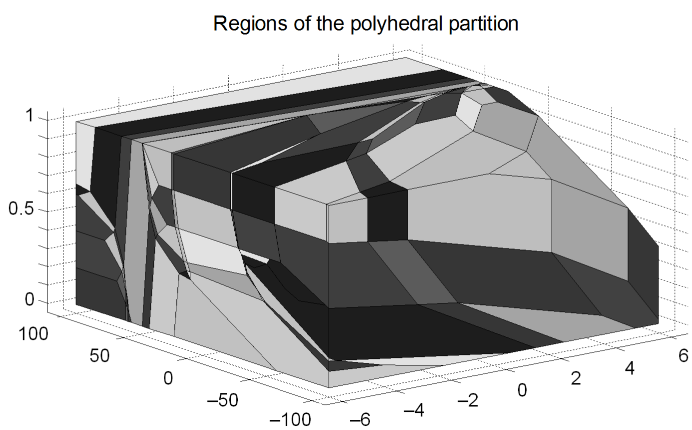

Fig. 3. Regions of the polyhedral partition

Simulation results considering the startup, step disturbance and input voltage step down of the brushed DC motor system are given in Figs. 3–6, where the results of EMPC controllers are drawn in solid lines and PI controllers in dashed lines.

The regions of the polyhedral partition are shown in Fig. 3. For each critical region, the optimal EMPC control law is an affine function of the states and the previous control input. Figure 4 gives the system trajectories during startup under EMPC and PI controllers. As we can see from the trajectories, the EMPC controller derives the angular speed to the reference quickly and with small overshot whereas the PI controller reacts slowly and results in big overshot. With the help of system constraint handling, EMPC control strategy respects the input and armature current constraint. Conventional PI controller deal with the input constraint by saturation but fail to handle armature current constraint. As far as the difference of two consecutive inputs is concerned, EMPC successfully restricts the difference within the constraint $[-0.2, 0.2]$. Figure 5 gives the state and input trajectories of the system in presence of angular speed step disturbance active for $t \geq 80$ s after startup. As we can see from the figures, the EMPC strategy settles down the system in presence of step disturbance quickly compared with the conventional PI schema. Figure 6 gives the closed-loop responses to the step-down change in the input voltage active for $t \geq 80$ s. It also shows that both EMPC and conventional PI can derive the angular speed to the reference, however, the deviations of angular speed and armature current is smaller under EMPC.

As we can see from the simulation results, EMPC improves the closed-loop performance systematically and the controller is easy to tune by adjusting the weight matrices.

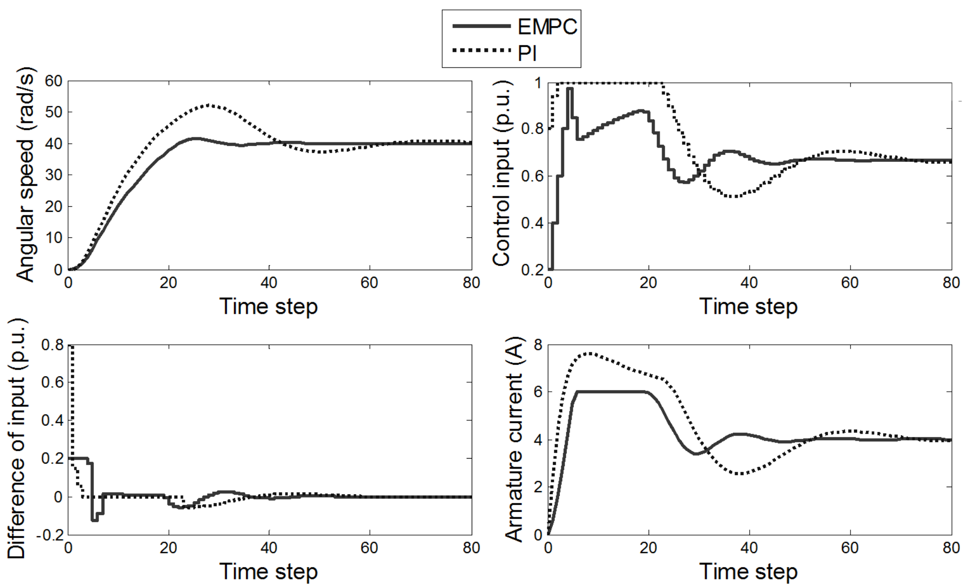

Fig. 4. Closed-loop responses during startup

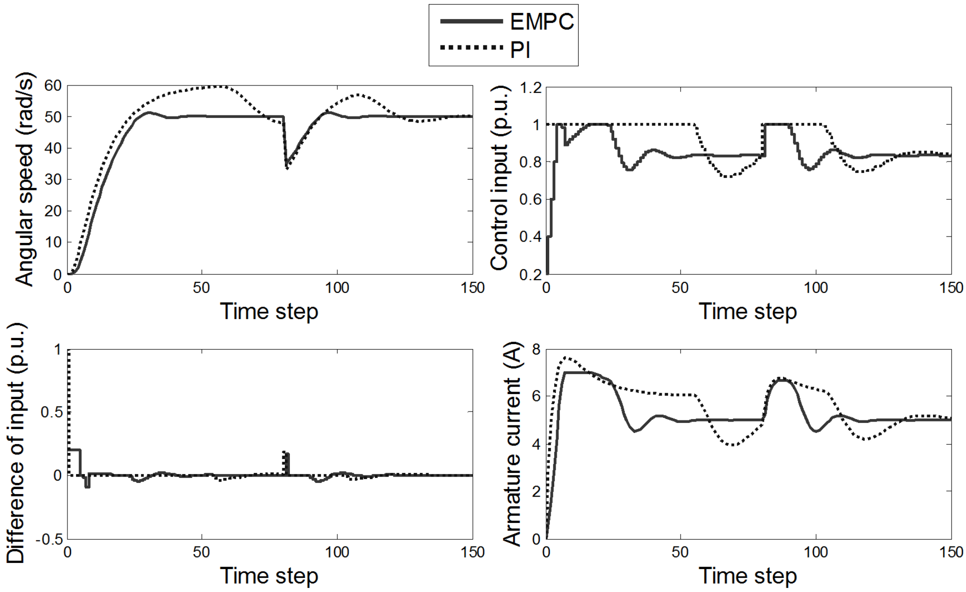

Fig. 5. Closed-loop responses to the step disturbance active for $t \geq 80\,\mathrm{s}$

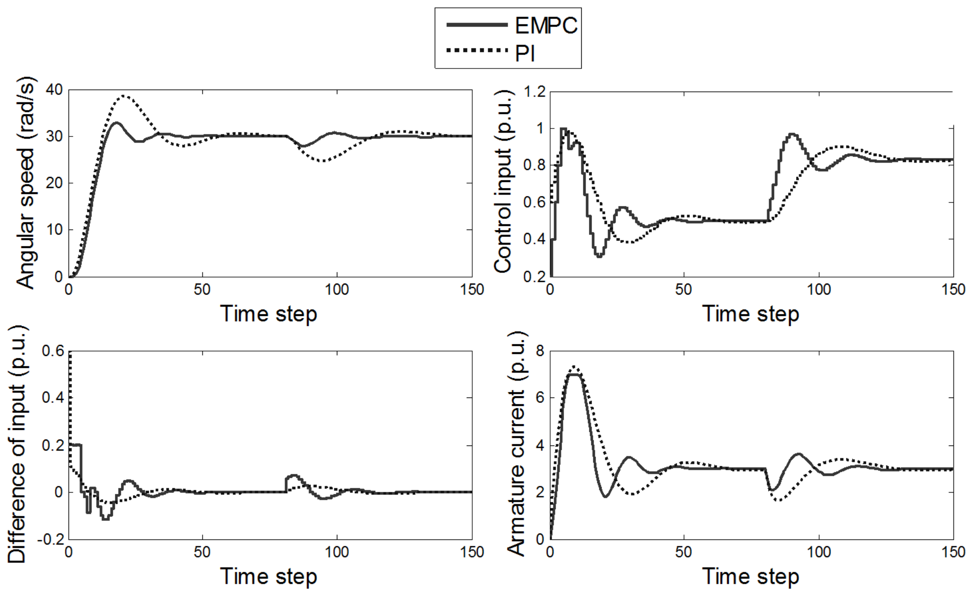

Fig. 6. Closed-loop responses to the step-down change in the input voltage active for $t \geq 80\,\mathrm{s}$

5. Conclusion

EMPC is proposed for brushed DC motor speed control under unmodelled disturbances. The EMPC strategy divides the state space into critical regions. For each critical region, the optimal control law is an affine function of the states and previous control input which could be calculated off-line. The constraint optimal speed control of brushed DC motor is transformed into a table look-up algorithm and the on-line computation time is greatly reduced. As a result, the brushed DC motor system is coordinately controlled and can easily be tuned by adjusting the weight matrices. EMPC improves the closed-loop performance remarkably compared with conventional PI regulator. Inspired by this benchmark example, EMPC can be extended to power electronic converters, electrical drives and related fields for constraint handling and optimal control. However, in order to apply this scheme to industry practice, a major and important work to be done is realization of the algorithm in the embedded system which has limited computation power and hardware resources.

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